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INTERTEMPORAL COMPETITIVE EQUILIBRIUM: A REAPPRAISAL OF A BASIC SOURCE OF INSTABILITY

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Abstract

This paper resumes a source of instability of intertemporal equilibrium which was anticipated by Garegnani (2003) and criticized by Schefold (2004). The author points out that a non orthodox tâtonnement pricing must be accepted if the theory has to be consistent with the Jevons's law of unique price. Such tâtonnement prescribes that the rule for adjusting the relative prices of commodities available at different times is different from the rule applied to the relative prices of contemporary commodities. The working of such a mechanism can be a fundamental source of instability of the intertemporal equilibria. This result seems to be a challenge for the stability of general equilibrium also in the context of more realistic non-tâtonnement disequilibrium processes.

J.E.L. Classification C62, D5, B13, B51

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Intertemporal Competitive Equilibrium: a Reappraisal of a Basic Source of Instability

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1. Introduction

This paper reconsiders a recent criticism to the theory of intertemporal equilibrium. Garegnani (2003) argues that aggregate (in terms of value) saving and investment functions belong to the determinants of an intertemporal equilibrium, and that the properties of such functions are a specific source of non meaningful equilibria, which is not subsumed under the traditional income or wealth effects. The quasi-equilibrium method adopted by Garegnani in his criticism is questionable and we share the objections which Schefold (2004) has addressed to it. However, we contend that Garegnani's criticism is valid independently of his quasi-equilibrium method. We shall reformulate the argument using a different approach which runs through the following analytical steps: i) a certain characterization of Jevons's law and the recognition of the existence of a uniform rate of return associated with an intertemporal equilibrium, ii) a

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reinterpretation of the model of individual behaviour underlying the excess demand functions, iii) the adoption of a tâtonnement pricing consistent with i)-ii), and iv) the analysis of the specific source of non meaningful equilibria which derives from the properties of the excess demand for capital flows. In particular, we shall explain why the auctioneer must follow a non orthodox tâtonnement, in which the rule for the adjustment of the relative prices of commodities available at different times is different from the rule applied to the relative prices of contemporary commodities. The working of such a mechanism will point out a basic source of instability. This result, which was anticipated by Garegnani (2003), seems to be a challenge for the stability of intertemporal general equilibrium, even in the context of more realistic non-tâtonnement analyses of disequilibrium.

2. Hahn's (Garegnani's) model of intertemporal equilibrium

Let us formulate a simplified version of the intertemporal equilibrium model which Hahn (1982) used in his criticism of the neo-Ricardians and which Garegnani (2003) resumed to introduce his criticism to the theory of intertemporal equilibrium. Our revision consists in assuming that i) each market is cleared in equilibrium by strictly positive prices and strict equality between demand and supply and ii) the linear techniques are given. The economy is assumed to exist for one period starting at time T = 0 and ending at time T = 1. The commodities of the economy are two non storable goods *a*, *b* available at times T = 0, 1 and labour performed during the period [0,1]. Let P_{a0}, P_{b0} be the prices of *a*, *b* available at T = 0; P_{a1}, P_{b1}, the prices of *a*, *b* available at T =1; W_1 the wage rate. All prices and the wage rate are nominal and wages are assumed to be paid at T = 1.

The price equations under perfect competition and constant returns to scale:

$$\left.\begin{aligned} P_{a1} &= l_a W_1 + a_a P_{a0} + b_a P_{b0} \\ P_{b1} &= l_b W_1 + a_b P_{a0} + b_b P_{b0} \end{aligned}\right| \qquad (1)$$

where l_a, l_b are given positive labour coefficients, a_a, b_a, a_b, b_b are given positive coefficients of goods a,b used as circulating capital. The technology is assumed to be viable.

Let $D_{jT}(\cdot)$, $j = a, b$, denote the demand function for consumption of goods *a*, *b* available at time $T = 0,1$ and $(\cdot)$ the relation with the independent variables $(P_{a0}, P_{b0}, P_{a1}, P_{b1}, W_1)$.

Market clearing equations for commodities available at T = 0:

$$\left.\begin{aligned} A_0 &= D_{a0}(\cdot) + (a_a A_1 + a_b B_1) \\ B_0 &= D_{b0}(\cdot) + (b_a A_1 + b_b B_1) \end{aligned}\right| \qquad (2)$$

where A_0, B_0 are <u>given endowments</u> of goods *a,b* at T = 0; A_1, B_1 are quantities of goods, *a*, *b* produced during the period and available for consumption at T = 1.

Labour market clearing:

$$l_a A_1 + l_b B_1 = L \qquad (3)$$

where L is a given supply of labour.

Market clearing equations for commodities available at time T =1:

$$\left.\begin{array}{l} A_1 = D_{a1}(\cdot) \\ B_1 = D_{b1}(\cdot) \end{array}\right| \qquad (4)$$

where $D_{a1}(\cdot), D_{b1}(\cdot)$ are demand functions for consumption of goods a, b at time T =1.

All demand functions are homogeneous of degree zero in $(P_{a0}, P_{b0}, W_1, P_{a1}, P_{b1})$ and satisfy Walras identity:

$$A_0 P_{a0} + B_0 P_{b0} + LW_1 \equiv D_{a0}(\cdot)P_{a0} + D_{b0}(\cdot)P_{b0} + D_{a1}(\cdot)P_{a1} + D_{b1}(\cdot)P_{b1} \qquad (5)$$

Let us assume good b available at T = 1 as the standard of value. The equation of price normalization

$$P_{b1} = 1. \qquad (6)$$

One equation among (1)-(4) depends on the others; therefore a solution to (1), (6), under the non negativity constraints, determines the quantities produced A_1, B_1 and the prices $P_{a0}, P_{b0}, P_{a1}, P_{b1}, W$. An equilibrium solution determines also the quantities consumed at T = 0,1, the quantities saved and invested at T = 0 and implies null quantities saved and invested at T = 1. The functions of aggregate saving $S_0(\cdot)$ and aggregate investment $I_0(\cdot)$ are defined:

$$
\left.
\begin{aligned}
&S_0(\cdot) \equiv [A_0 - D_{a0}(\cdot)]P_{a0} + [B_0 - D_{b0}(\cdot)]P_{b0} \\
&\mathrm{I}_0(\cdot) \equiv [a_a D_{a1}(\cdot) + a_b D_{b1}(\cdot)]P_{a0} + [b_a D_{a1}(\cdot) + b_b D_{b1}(\cdot)]P_{b0} \, .
\end{aligned}
\right| \quad (7)
$$

$S_0(\cdot)$, $I_0(\cdot)$ can serve for the interpretation of an equilibrium, but they seem to play no distinct causal role for the determination of the equilibrium itself, in comparison with the demand and supply functions of the individual physical commodities.[1]

3. Orthodox tâtonnement applied to a reduced form

Let us substitute the quantities A_1, B_1 in (2), (3) with equations (4) and the prices P_{a1}, P_{b1} in the demand function , $D_{jT}(\cdot)$, $j = a,b$; $T = 0,1$, with the price equations (1). Let $\mathbf{p} = (P_{a0}, P_{b0}, W_1)$ denote the price vector and $d_{jT}(\mathbf{p})$ the demand function after substitution of P_{a1}, P_{b1} in $D_{jT}(\cdot)$. The corresponding excess demand functions are:

$$
\begin{aligned}
&E_{a0}(\mathbf{p}) \equiv d_{a0}(\mathbf{p}) + a_a d_{a1}(\mathbf{p}) + a_b d_{b1}(\mathbf{p}) - A_0 \\
&E_{b0}(\mathbf{p}) \equiv d_{b0}(\mathbf{p}) + b_a d_{a1}(\mathbf{p}) + b_b d_{b1}(\mathbf{p}) - B_0 \\
&E_L(\mathbf{p}) \equiv l_a d_{a1}(\mathbf{p}) + l_b d_{b1}(\mathbf{p}) - L \, .
\end{aligned}
$$

[1] This negative remark has been put forward by Schefold (2004).

The E functions are homogenous of degree zero in $\mathbf{p}$ and satisfy the following identity, derived from Walras law (5) and from the assumption that the markets for consumption goods at time T = 1 are in equilibrium (equation (4)):

$$P_{a0}E_{a0}(\mathbf{p}) + P_{b0}E_{b0}(\mathbf{p}) + W_1E_L(\mathbf{p}) \equiv 0 \qquad (8).$$

Let us remain within the limits of adjustment processes with tâtonnement and let t denote the logical time attached to the iterations performed by the Walrasian auctioneer. The typical difficulty of tâtonnement for a production economy, under constant returns to scale, is "solved" here by transforming model (1), (6) into a reduced form to which tâtonnement pricing is applied. This amounts to the method of calling prices suggested by Schefold (2003).[2] In our case, such tâtonnement analysis becomes a quasi-equilbrium analysis also because the markets for consumption goods at time T = 1 are assumed to be always in equilibrium.

Let us suppose that $H_a(E_{a0}(\mathbf{p}))$, $H_b(E_{b0}(\mathbf{p}))$, $H_L(E_L(\mathbf{p}))$ are sign-preserving functions of the excess demands, with $H_a(0) = H_b(0) = H_L(0)$ = 0. The following differential equations[3] describe an orthodox dynamics with

[2] At each iteration the auctioneer is supposed to call only the prices of the initial endowments and to receive back from the producers the information of the prices of goods a, b at T = 1, which satisfy (1) and, in a more general model with alternative methods of production, are associated with the choice of the cost minimizing techniques; next the auctioneer receives the information of the individual net demands and calculates the corresponding aggregate excess demands in order to call new prices of the initial endowments.

[3] As we are not concerned with proofs of stability, we have not specified other properties of the functions E and H (in particular the usual assumption that the functions are continuos and differentiable) and the assumption that assures that, if $\mathbf{p}(0) > 0$, then $\mathbf{p}(t) > 0$ for all $t < +\infty$.

tâtonnement:

$$\left.\begin{aligned} \frac{dP_{a0}}{dt} &= H_a\left(E_{a0}(\mathbf{p})\right) \\ \frac{dP_{b0}}{dt} &= H_b\left(E_{b0}(\mathbf{p})\right) \\ \frac{dW_1}{dt} &= H_L\left(E_L(\mathbf{p})\right) \end{aligned}\right| \qquad (9)$$

where the prices satisfy the equations $P_{b1} = l_b W_1 + a_b P_{a0} + b_b P_{b0} = 1$ and the functions $H_a(\cdot), H_b(\cdot), H_L(\cdot)$ satisfy the identity $a_b H_a(\cdot) + b_b H_b(\cdot) + l_b H_L(\cdot) \equiv 0$. Then, given the initial call of prices, $\mathbf{p}(0)$, a path of subsequent calls $\mathbf{p}(t)$, $t > 0$, can be determined by any two differential equations chosen from (9) and by the numeraire equation.

Therefore, the determination not only of an equilibrium solution, but also of the stability properties of such tâtonnement rule leaves no "causal" role to the functions $S_0(\cdot)$, $I_0(\cdot)$, of aggregate saving and investment. We may concede that, granted the validity of Walras law (5), we could replace one equation chosen from (1)-(4) with the equation $S_0(\cdot) = I_0(\cdot)$ to calculate an equilibrium solution. This substitution does not leads us very far, because it does not assign to $S_0(\cdot)$, $I_0(\cdot)$ any special role in the adjustment mechanism.

However the auctioneer, instead of crying prices according to equations (9), is compelled to follow a different rule, if the theory is consistent with the extension of Jevons's Law to the capital goods.

4. The existence of a uniform effective rate of return

Assume that in equilibrium $P_{a0} > 0$, $P_{b0} > 0$, $P_{a1} > 0$, $P_{b1} > 0$, $W_1 > 0$ and define the own rates of interests $r_a \equiv \frac{P_{a0}}{P_{a1}} - 1$, $r_b \equiv \frac{P_{b0}}{P_{b1}} - 1$. Let $\hat{p}_{a0} = \frac{P_{a0}}{P_{b0}}$, $\hat{p}_{a1} = \frac{P_{a1}}{P_{b1}}$, $\hat{p}_{b0} = \frac{P_{b0}}{P_{b0}} = 1$, $\hat{p}_{b1} = \frac{P_{b1}}{P_{b1}} = 1$, $\hat{w}_1 = \frac{W_1}{P_{b1}}$ denote current relative prices with good *b* chosen as a dated numeraire.

The price equations at current prices:

$$\left.\begin{aligned} \hat{p}_{a1} &= l_a \hat{w}_1 + (a_a \hat{p}_{a0} + b_a \hat{p}_{b0})(1 + r_b) \\ \hat{p}_{b1} &= l_b \hat{w}_1 + (a_b \hat{p}_{a0} + b_b \hat{p}_{b0})(1 + r_b) \end{aligned}\right| \qquad (10)$$

Equations (10) can be written:

$$\left.\begin{aligned} \left(\frac{\hat{p}_{a1} - l_a \hat{w}_1}{\hat{p}_{a0} - l_a \hat{w}_0}\right)\left(\frac{\hat{p}_{a0} - l_a \hat{w}_0}{a_a \hat{p}_{a0} + b_a \hat{p}_{b0}}\right) &= 1 + r_b \\ \left(\frac{\hat{p}_{b1} - l_b \hat{w}_1}{\hat{p}_{b0} - l_b \hat{w}_0}\right)\left(\frac{\hat{p}_{b0} - l_b \hat{w}_0}{a_b \hat{p}_{a0} + b_b \hat{p}_{b0}}\right) &= 1 + r_b \end{aligned}\right| \qquad (10')$$

where $w_0 \equiv \dfrac{\hat{w}_1}{1+r_b}$ is the discounted wage rate. Each equationn (10’) sets that the effective factor of return, received from investing in the production of a certain good, is equal to $1+r_b$, the own factor of interest on the numeraire. The “effectiveness” of such return results from the multiplication of the terms in brackets: 1) the factor of appreciation of a bundle of a good and of a ‘bad’ (i.e. a labour coefficient), 2) the own factor of profit calculated at contemporary prices.

The law of unique price imposes the relation:

$$\frac{\hat{p}_{a1}}{\hat{p}_{a0}}(1+r_a) = \frac{\hat{p}_{b1}}{\hat{p}_{b0}}(1+r_b) \qquad (11)$$

between the effective factors of return received from saving and lending the goods a, b. The effective rate of return on each good is calculated by multiplying its own factor of interest for the factor of appreciation of the good itself. It should be noted that (11) pertains to the sphere of exchange and must be interpreted as an equation, although its mathematical form resembles a tautology. The usual notation of the relative prices, written as ratios between nominal prices, can be indeed misleading. In fact (11) can be written

$$\left(\frac{P_{a1}}{P_{b1}}\right)\cdot\left(\frac{P_{b0}}{P_{a0}}\right)\cdot\left(\frac{P_{a0}}{P_{a1}}\right) = \frac{P_{bo}}{P_{b1}} \qquad (11')$$

which apparently seems to be an identity. This is not the correct interpretation of (11). The relative price associated with a direct exchange of two commodities is not equal by definition to the corresponding relative price which is implicit in a chain of (e.g. triangular) exchanges involving other commodities. In particular, $\left(\frac{P_{b0}}{P_{b1}}\right)^{implicit} \equiv \left(\frac{P_{a1}}{P_{b1}}\right)\cdot\left(\frac{P_{b0}}{P_{a0}}\right)\cdot\left(\frac{P_{a0}}{P_{a1}}\right)$ is an implicit relative price associated with three binary exchanges of goods; instead $\frac{P_{b0}}{P_{b1}}$ is a direct exchange ratio. Hence (11) or (11') or $\left(\frac{P_{b0}}{P_{b1}}\right)^{implicit} = \left(\frac{P_{b0}}{P_{b1}}\right)$ are equations. They must be interpreted as an application of Jevons's law (the law of unique price), which we assume to hold both in equilibrium and in disequilibrium. They are the outcome of spot-forward arbitrages on goods a, b, and they might be violated if the markets should not be perfect.

Since good b is the numeraire , equation (11) becomes

$$\frac{\hat{p}_{a1}}{\hat{p}_{a0}}\left(1+r_a\right)=1+r_b\,. \qquad (11'')$$

It follows from (10'), (11'') that r_b represents the uniform effective rate of return, which in the model applies to saving, lending and productive investment. The recognition of a uniform rate of return for an economy, which is not in a long period equilibrium, is a crucial step of the argument.

Garegnani assert 1) that capital goods are perfect substitutes for the saver and 2) that the properties of the relative prices of commodities available at different times (intertemporal relative prices) are different from those pertaining to the relative prices of commodities available at the same time (contemporary or current relative prices). In the theory of intertemporal equilibrium 1) and 2) must be grounded on an explicit model of individual choice. We shall isolate such a model through some intermediate steps, which aim to avoid certain possible misunderstanding.

4. Capital goods are perfect substitutes for the saver

Following Schefold's (2004) reconstruction of the microfoundations of Hahn's (Garegnani) model, let us assume that the demand functions of model (1)-(6) are derived from the rational choices of a representative consumer, who is supposed to solve the problem

$$\left.\begin{array}{c} \max\ u(\cdot) \\ \text{s.t.} \\ a_0 P_{a0} + b_0 P_{b0} + \mathit{l} W_1 = c_{a0} P_{a0} + c_{b0} P_{b0} + c_{a1} P_{a1} + c_{b1} P_{b1}\,, \end{array}\right| \qquad (12)$$

that is the intertemporal budget constraint, where a_0, b_0 are the initial endowments of goods a, b ; l the labour endowment; c_{jT} the dated consumption of good j, $j = a,b$, T = 0,1; and $u(\cdot)$ is the utility function, all notations referred to the consumer.

In microeconomics the attribute "perfect substitutes" has a definite meaning in the following cases:

i) if we specify $u(\cdot) = u(c_{a0}, c_{b0}, c_{a1}, c_{b1})$ and we assume that the marginal rate of substitution between two consumption goods is constant (perfect substitutes);

ii) if we define the indirect utility function $f\left(P_{a0}, P_{b0}, P_{a1}, P_{b1}, W_1, y\right) \equiv \max u\left(c_{a0}, c_{b0}, c_{a1}, c_{b1}\right)$ s.t. $y = c_{a0}P_{a0} + c_{b0}P_{b0} + c_{a1}P_{a1} + c_{b1}P_{b1}$,where $y \equiv a_0 P_{a0} + b_0 P_{b0} + lW_1$ and we say that the physical constituents of y are perfect substitutes;

iii) if we assume that the consumer postpones at time T = 1 the choice of consumption goods at T =1 and we specify $u(\cdot) = u(c_{a0}, c_{b0}, s)$ where $s = (a_0 - c_{a0})P_{a0} + (b_0 - c_{b0})P_{b0}$ is his total saving at discounted prices. In this case we can say that the physical constituents of s are perfect substitutes.

The attribute "perfect substitutes" in the intertemporal context at issue does not fit in any of cases i), ii), iii) above. In particular, the model (1)-(6) rules out case iii), because the choice of the whole intertemporal plan of consumption and saving is supposed to be made only at time T = 0 and therefore the rational consumer does not attach any utility to the degrees of freedom (flexibility, liquidity) of choice at time T = 1. Still cases ii) and iii) suggest that another reason exists for the saver to treat the capital goods as perfect substitutes, despite the fact that they do not enter into the (direct or indirect) utility function of the consumer. In fact, we may distinguish different facets, roles, functions of the same representative decision maker: he is a consumer, a saver, an investor and a worker at the same time. Each facet can be supposed to maximize some objective function and the ensuing (non schizophrenic) result is an optimal plan of consumption $c^*_{a0}, c^*_{b0}, c^*_{a1}, c^*_{a1}$, as a solution to problem (12). Besides the signals of the net demands for goods and of the supply of labour services which are sent by the consumer,

$$c_{a0}^{*} - a_{0}\,,\ \ c_{b0}^{*} - b_{0}\,,\ \ c_{a1}^{*}\,,\ c_{a1}^{*}\,,\ \ l^{*} = l\ ,$$

the saver sends the signal of his optimal saving plan at time T = 0 (with no saving at T = 1):

$$s^{*} = (a_{0} - c_{a0}^{*})P_{a0} + (b_{0} - c_{b0}^{*})P_{b0}\,.$$

It remains to explain why s^{*} can be an independent effective signal. Why, six signals, instead of five, are received by the auctioneer as distinct effective market signals?

We can imagine that the saver receives from the consumer (the same individual) the quantities non consumed $(a_{0} - c_{a0}),\ (b_{0} - c_{b0})$ and the purpose of the former is to transform their value $s\ = (a_{0} - c_{a0})P_{a0} + (b_{0} - c_{b0})P_{b0}$ into the maximum purchasing power available at time T = 1. As in case ii), the physical constituents of s are perfect substitutes for him, at the given contemporary prices P_{a0}, P_{b0}. He would change the physical composition $(a_{0} - c_{a0})$, $(b_{0} - c_{b0})$ of s before lending the goods, if all contemporary arbitrages should not be fully exploited; otherwise he is indifferent to the basket of goods contained in s. For the same reason the physical constituents of the income that he received from the borrowers at the end of the period are perfect substitutes for the saver. He would exploit all possible spot-forward arbitrages in case equation (11”) should not be initially satisfied, otherwise he is indifferent to the physical composition of the income that he receives from the borrowers at T = 1. We should note that for the saver “perfect substitutability” implies indifference in the choice of the physical mix of saving only if the prices given to him satisfy Jevons's law expressed by equation (11"), which concerns the effective own rates of return. Of course, saying that the goods *a*, *b* are perfect substitutes for the saver does not mean that they are such also for the consumer. Each dated good *a*, *b* is physically homogeneous; yet, from the point of view of the two facets of the individual, $c_{a0}, c_{b0}, c_{a1}, c_{b1}$ are consumption goods, whereas $(a_0 - c_{a0})$, $(b_0 - c_{b0})$ are capital goods.

5. Different properties of two types of relative prices and heterodox tâtonnement

The previous argument implies that the auctioneer is prevented from controlling each own rate of return independently from the others. At each iteration he cannot cry prices which violate equations (10') and (11"). A change in r_b drags up or down all equalized own effective rates of return and cannot be directly affected by the physical excess demand for capital good *b*. Only the sign of the total value of the excess demands for all capital goods at time T = 0 can induce the auctioneer to change the uniform effective rate of return in a definite direction. This brings about a quite different tâtonnement, compared to the orthodox model (9).

Now the auctioneer is assumed to follow different rules for changing prices, according to the distinction between current (contemporary) relative prices and intertemporal relative prices. It is convenient to reformulate the excess demand as functions of current prices and of the uniform effective rate of return.

The individual budget constraint at current prices with $\hat{p}_{b0} = 1$, $\hat{p}_{b1} = 1$ is:

$$a_a \hat{p}_{a0} + b_0 + \frac{l\hat{w}_1}{1+r_b} = c_{a0}\hat{p}_{a0} + c_{b0} + \frac{c_{a1}\hat{p}_{a1}}{1+r_b} + \frac{c_{b1}}{1+r_b} \quad . \qquad (13)$$

The aggregate demand function for commodity j at time T: $\tilde{D}_{jT}\left(\hat{p}_{a0}, \hat{p}_{a1}, \hat{w}_1, r_b\right)$. We can replace two prices in $\tilde{D}_{jT}(\cdot)$ with a solution to the price equations (10), provided that r_b falls within its feasible range. In particular, after substitution of $\hat{p}_{a1}$ and $\hat{w}_1$, we obtain the demand function $\tilde{d}_{jT}\left(\hat{p}_{a0}, r_b\right)$. Let us define the price vector $\hat{\mathbf{p}} = \left(\hat{p}_{a0}, r_b\right)$ and the excess demand functions:

$$\begin{aligned}
&\tilde{E}_{a0}(\hat{\mathbf{p}}) \equiv \tilde{d}_{a0}(\hat{\mathbf{p}}) + a_a\tilde{d}_{a1}(\hat{\mathbf{p}}) + a_b\tilde{d}_{b1}(\hat{\mathbf{p}}) - A_0 \\
&\tilde{E}_{b0}(\hat{\mathbf{p}}) \equiv \tilde{d}_{b0}(\hat{\mathbf{p}}) + b_a\tilde{d}_{a1}(\hat{\mathbf{p}}) + b_b\tilde{d}_{b1}(\hat{\mathbf{p}}) - B_0 \\
&\tilde{E}_{L}(\hat{\mathbf{p}}) \equiv l_a\tilde{d}_{a1}(\hat{\mathbf{p}}) + l_b\tilde{d}_{b1}(\hat{\mathbf{p}}) - L
\end{aligned}$$

The $\tilde{E}$ functions satisfy Walras law $P_{a0}\tilde{E}_{a0}(\hat{\mathbf{p}}) + P_{b0}\tilde{E}_{b0}(\hat{\mathbf{p}}) + W_1\tilde{E}_L(\hat{\mathbf{p}}) \equiv 0$, but they are not homogenous of degree zero in $\hat{\mathbf{p}} = \left(\hat{p}_{a0}, r_b\right)$.

In the alternative reduced form of model (1),(6), we have one contemporary relative price, $\hat{p}_{a0}$, and one intertemporal relative price, the rate r_b. The auctioneer will call a higher (lower) $\hat{p}_{a0}$ if and only if he finds a positive (negative) excess physical demand $\tilde{E}_{a0}(\hat{\mathbf{p}})$. Instead he will call a higher (lower) rate r_b if and only if he finds <u>that the value of the aggregate demand for investment exceeds (falls short of) the value of the aggregate supply of saving</u>. [4]. Such aggregate excess demand at current values is $\hat{p}_{ao}\tilde{E}_{a0}(\hat{\mathbf{p}}) + \hat{p}_{bo}\tilde{E}_{b0}(\hat{\mathbf{p}})$. Let $\tilde{H}_a, \tilde{H}_b$ be smooth sign-preserving functions of excess demands, with $\tilde{H}_a(0) = \tilde{H}_b(0) = 0$. Then the following differential equations determine the tâtonnement dynamics for the whole economy:

$$
\begin{aligned}
\frac{d\hat{p}_{a0}}{dt} &= \tilde{H}_a\left(\tilde{E}_{a0}(\hat{\mathbf{p}})\right) \\
\frac{dr_b}{dt} &= \tilde{H}_b\left(\hat{p}_{a0}\tilde{E}_{a0}(\hat{\mathbf{p}}) + \tilde{E}_{b0}(\hat{\mathbf{p}})\right)
\end{aligned}
\qquad (14)
$$

Given the initial prices, $\hat{\mathbf{p}}(t)$, $t = 0$, a path $\hat{\mathbf{p}}(t)$, $t > 0$ is determined by the differential equations (14). The paths of the remaining current and intertemporal prices follow from the relations between current prices and discounted prices and from the price equations (1) , (10)with the numeraire equations $\hat{p}_{b1} = 1$, and $P_{b1} = 1$. The corresponding path of the excess demand for labour $\tilde{E}_L(\hat{\mathbf{p}})$ follows from Walras law

[4] Assuming that the price of a certain commodity reacts not only to the excess demand for that commodity but also to the excess demand for other commodities is not a novel approach in stability analysis. In particular, the application of Newton's method of numerical analysis would prescribe that the price of each commodity reacts to the excess demand for all commodities by a certain uniform coefficient and brings about a proportional decrease in all excess demands (see Smale (1976)). However, the specific feature of the adjustment described by (14) is the fact that an intertemporal price reacts to the <u>value</u> of the excess demands for capital flows.

$$P_{a0}\tilde{E}_{a0}(\hat{\mathbf{p}}) + P_{b0}\tilde{E}_{b0}(\hat{\mathbf{p}}) + W_1\tilde{E}_L(\hat{\mathbf{p}}) \equiv 0 .$$

The heterodox system of differential equations (14) can be compared with the orthodox system (9). The excess aggregate investment over aggregate saving plays a causal role in the adjustment process (14). As a consequence, the standard stability analysis is subverted. In particular, even the strong assumption that all physical goods are gross substitutes, with respect to the functions of the excess demands $E_{a0}(\mathbf{p}), E_{b0}(\mathbf{p}), E_L(\mathbf{p})$, does not imply that the same property applies to the physical excess demands $\tilde{E}_{a0}(\hat{\mathbf{p}})$, $\tilde{E}_{b0}(\hat{\mathbf{p}})$, $\tilde{E}_L(\hat{\mathbf{p}})$ and to the aggregate excess demand for the capital flow $\hat{p}_{a0}\tilde{E}_{a0}(\hat{\mathbf{p}}) + \tilde{E}_{b0}(\hat{\mathbf{p}})$, which are the relevant functions in the tâtonnement equations (14).

Therefore the problem of the shape of the demand and supply schedules of aggregate capital re-emerges, although the model is formulated by taking the physical endowments, instead of their total value, as given. This is the main point stressed by Garegnani. The present case is different from the problem of capital in the controversies of the Sixties only because the dimensions of the magnitudes at issue are different: the flow dimension of investment and saving versus the stock dimension of the demand and supply of capital. 'Badly behaved' aggregate saving and investment functions can be met in a model with many heterogenous capital goods and can become a specific source of unstable equilibria for the same reasons. In particular, the assumption that all goods are gross substitutes or the assumption of diagonal dominance (jointly with a minor property imposed on the excess supply of the numeraire)[5], with respect to the excess demands functions $E_{a0}(\mathbf{p}), E_{b0}(\mathbf{p}), E_L(\mathbf{p})$, are still sufficient conditions for the uniqueness of equilibrium, but the theorems of global stability which rest upon them do not prove that equilibrium is stable with respect to the heterodox tâtonnement rule (14) which is consistent with Jevons' Law.

6. Final comment and conclusion

Let us summarize the main results of the previous analysis and certain unsettled questions.

1) The role of aggregate capital flows emerges from the heterodox analysis of stability which has been formulated by equations (14), despite it is absent in the determination of an equilibrium solution to equations (1), (6) and in the traditional tâtonnement (9).
2) That role does not presuppose a monetary economy under uncertainty[6] and that type of aggregation does not put an additional threat to methodological individualism, in comparison to that which already derives from the rudimentary price dynamics based on the assumption of the auctioneer which calls unique prices.

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[5] See Arrow-Hahn (1971, Ch.9)

[6] In this respect we depart from the interpretation of Garegnani's approach given by Schefold (2004), although the latter correctly points out that a similarity exists between the approach adopted by the former and Clower's (1969) formulation of the microeconomic foundations of Keynes's aggregate demand function.

3) In the simple model of intertemporal equilibrium (1)-(6), production is subjected to constant returns to scale and the auctioneer is supposed to be constrained in calling prices according to a rule similar to that adopted by Schefold (2003). The intertemporal equilibrium model and the corresponding tâtonnement could be extended assuming that alternative techniques are available.

4) We argued that the heterodox tâtonnement (14) brings about a subversion in stability analysis. The general conclusion is that an intertemporal equilibrium becomes exposed to a specific source of instability (Garegnani 2003). We should refrain from concluding that, since the new tâtonnement process may be unstable even under strong restrictions put on the excess demand functions for individual goods (the assumption of gross substitutes), a-fortiori more complicated adjustment processes (e.g. processes with trading at false prices) may be unstable as well. Further investigations should be necessary in this field of analysis, because some counterintuitive results may turn out, as already Franklin Fisher (1983) has reminded us with regard to the passage from the traditional tâtonnement to more realistic stability processes, where the convergence depends not only on the properties of the excess demand functions, E, but also on those of the reactions functions, H. However, also non-tâtonnement disequilibrioum models should be monitored in order to ascertain whether they are exposed to the fundamental source of instability which has been re-appraised in this paper.

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